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Treatment of a relativistic particle in external electromagnetic field as a singular system(*)

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Summary. — A relativistic particle in external electromagnetic field is studied by the canonical method. Two examples are studied in detail.

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1. - Introduction

The Hamiltonian formulation of constrained systems is first discussed by Dirac [1, 2]. If the rank of the Hessian matrix

$$\frac{\partial^2 L}{\partial \dot{q}_i \partial \dot{q}_j}\,, \qquad i, j = 1 \dots n\,, \tag{1.1}$$

is $n - r$, $r < n$, then one can produce r functionally independent constraints,

$$H'_\mu(q, p) \approx 0\,, \qquad \mu = 1, \dots, r \tag{1.2}$$

which are called primary constraints. The total Hamiltonian is defined as

$$H_{\mathrm{T}} = H_0 + v_\mu H'_\mu\,, \tag{1.3}$$

where H_0 is the standard Hamiltonian

$$H_0 = -L + p_i \dot{q}_i \tag{1.4}$$

and v_μ are coefficients (Einstein summation rule is used throughout this paper).

Consistency conditions are given as

$$\dot{H}'_\mu = \frac{\mathrm{d}H'_\mu}{\mathrm{d}t} = \{H'_\mu, H_0\} + v_\varrho \{H'_\mu, H'_\varrho\} \approx 0\,. \tag{1.5}$$

(*) The authors of this paper have agreed to not receive the proofs for correction.

These conditions may be identically satisfied as a result of the primary constraints, or they lead to new conditions which are called the secondary constraints. Primary and secondary constraints can be classified as first class and second class. First-class constraints are those which have vanishing Poisson brackets with all other constraints and second-class constraints are those which have non-vanishing Poisson brackets. Second-class constraints could be used to eliminate some of p's and q's from the theory.

Recently, a second method which is called the canonical method is introduced [3-7]. The equivalent Lagrangians method is used to obtain the set of Hamilton-Jacobi partial differential equations (HJPDE) as

$$H'_\alpha \left(t_\beta, q_a, \frac{\partial S}{\partial q_a}, \frac{\partial S}{\partial t_\beta} \right) = 0, \qquad \alpha, \beta = 0, 1, \ldots, r, \qquad a = 1, \ldots, n - r, \tag{1.6}$$

where

$$H'_\alpha = H_\alpha + P_\alpha . \tag{1.7}$$

The equations of motion are given as total differential equations in variables t_β,

$$\mathrm{d}q_a = \frac{\partial H'_\alpha}{\partial p_a} \mathrm{d}t_\alpha , \qquad \mathrm{d}p_a = - \frac{\partial H'_\alpha}{\partial q_a} \mathrm{d}t_\alpha , \tag{1.8}$$

$$\mathrm{d}p_\mu = - \frac{\partial H'_\alpha}{\partial q_\mu} \mathrm{d}t_\alpha , \qquad \mathrm{d}z = \left(-H_\alpha + p_a \frac{\partial H'_\alpha}{\partial p_a} \right) \mathrm{d}t_\alpha , \tag{1.9}$$

where

$$Z = S(t_\alpha, q_a) . \tag{1.10}$$

Since equations are total differential equations, integrability conditions should be checked. As is mentioned in previous publications, equations of motion are integrable if variations of H'_α vanish identically. If they do not vanish identically we consider them as new constraints. This procedure continues until a complete system is obtained.

The validity of the canonical method has been tested in previous publications. In this paper we would like to apply the same method to the motion of a relativistic particle in external magnetic field.

This paper is arranged as follows. In the sect. **1** the canonical method is described briefly. Equations of motion are set up in sect. **2** and integrability conditions are tested. In sect. **3** examples are studied and results are discussed in sect. **4**.

2. – Motion of a relativistic charged particle

The motion of a relativistic particle of charge e in external electromagnetic field is described by the singular Lagrangian

$$L = -mc \sqrt{g^{ab} \dot{q}_\alpha \dot{q}_\beta} - \frac{e}{c} \dot{q}_\alpha A_\alpha , \qquad \alpha, \beta = 0, 1, 2, 3 . \tag{2.1}$$